THE MANY-OBJECT "PILOT WAVE" THEORY

Yoav Ben-Dov.
Institute for the History and
Philosophy of Science and Ideas.
Tel-Aviv University
69 978 Israel

Abstract

The "pilot wave" supplementary variables version of quantum mechanics is discussed. It is claimed that in the many-object case, a semi-classical picture of particles "guided" in their motion by waves in 3-space is difficult to maintain. Other interpretive schemes are suggested.

The Single-Object Case

At first sight, the "pilot wave" theory - historically connected with the names of de Broglie (1927) and Bohm (1952), and undoubtedly the most interesting "supplementary variables" version of non-relativistic quantum mechanics - may look quite simple when applied to the one-particle case. Consider the following recipe: Take a classical "particle", that is, a physical object with mass m whose location x is well-defined as a smooth function of time t. In addition to $x(t)$, also assign to the "particle" a complex quantum wave function $\Phi(r,\ t) = A(r,\ t)e^{is(r,\ t)}$, which is assumed always to develop according to the Schrödinger equation. Now add the following two assumptions:

1. The probability at time t of x being at some point r of 3-space is

$$Pr.[x(t) = r] = A(r,\ t)^2. \qquad (1)$$

2. The first time derivative ("velocity") of $x(t)$ is given by

$$dx/dt = (1/m) \nabla S(x,\, t). \tag{2}$$

The two assumptions given here are mutually consistent, in the sense that if Eq. (1) is posed as an initial condition for an ensemble of objects having the same Φ associated with each one of them, and whose velocities are given by Eq. (2), then Eq. (1) remains valid for that ensemble at any later t. Also, if we further assume that $x(t)$ gives the actually observed results of position measurements made on the object, then Eq. (1) guarantees the exact reproduction of the usual quantum predictions for the results of such measurements. Thus, it seems that we have a cheap way (perhaps "too cheap" see Einstein 1952) of reproducing the observed quantum predictions, while keeping to an almost–classical framework. This semi–classical view of the formalism is further made plausible by the fact that Eq. (2) is consistent (in a sense similar to that above, that is, as a condition whose validity is conserved by a second–order Newtonian equation) with the usual Hamiltonian formulation of classical mechanics, provided that the usual "classical" potential is supplemented by an additional "quantum potential" term:

$$Q(r,\, t) = -(^2/2m) \nabla^2 A(r,\, t)/(r,\, t) \tag{3}$$

so that the passage from classical to quantum mechanics appears to involve just the addition of some new "quantum force field" into the old framework.

But the simplicity of this appearance is somewhat misleading. Even at this stage, the new term $Q(r,\, t)$ exhibits some strange features which are not shared by classical potentials. For example, as Bohm points out, if the physical field responsible for the new quantum effects is taken to be the wave function Φ itself, then its influence on the dynamics through $Q(r,\, t)$ depends only on the field form, and not on its intensity. In addition, if we try to associate in the usual manner kinetic energy and momentum with the "object velocity" given in Eq. (2), then in the general case, these are not conserved because the action of the Φ field on the $x-$ coordinate is not counter balanced by any reaction; also, these violently fluctuating values have no relation whatsoever to the measured values of energy and momentum, which are given as usual by the eigenvalues of Φ (de Broglie 1930). Of course, one may restore energy conservation by counting the quantum potential term as an additional energy; but this does not solve the problem for the momentum.

The Many–Object Case

The semi–classical "quantum potential" picture presented in the previous section becomes more difficult to maintain once we turn to consider several interacting objects. Already Pauli (1927), in his criticism of de Broglie's presentation of the "pilot wave" theory at the fifth Solvay conference, used such an example (namely, a scattering of a free particle from a Fermi rotator) to show that after the interaction, a single object treatment in the "pilot wave" picture does not seem to give sensible results.

Although in his brief reply to Pauli, de Broglie did mention the difference between his own presentation of the single–object case and Pauli's two–object example, the detailed answer to Pauli's argument was given only many years later by Bohm (1952). As Bohm shows, the difficulty pointed out by Pauli disappears once the case is treated within a generalization of the "pilot wave" formalism to the 6-dimensional configuration space of the two interacting objects. In this article, Bohm refers to the two–object wave function as "a six-dimensional but objectively real field" an expression which de Broglie himself, who always refused to attach a physical meaning to the configuration space, could never accept (see Ben–Dov 1989).

Obviously, once we start to consider more complicated cases which include, for example, measurement set–ups (eventually including the human observer treated as a physical system), such a generalization has to be carried on to the $3N$-dimensional configuration space of all the interacting objects. Thus, we are led to Bell's (1980) conclusion that "the correct application of the theory is to the world as a whole" in which "the world" is taken to include all the mutually interacting objects, including measuring apparatus and even human observers.

In a way which basically follows Bell's approach (see also Ben–Dov 1987), we shall now present the mathematical formalism of the "pilot wave" theory for the N-object case, of which the single-object formalism can be regarded as a special case. Let $r_1 \ldots r_N$ be the space coordinates of the N objects. We make the following four assumptions:

(MO1) With the complete system is associated a quantum wave function $\Phi(r_1 \ldots r_N,\ t)$, which always develops according to the Schrödinger equation and never "collapses".

Here, we disregard "internal" degrees of freedom such as spin. As Bell (1981) shows, these may be accounted for by considering a multi-component

Φ, although one may also think of other possibilities, for example the "vortex" spin model suggested by Bohm et al (1955).

(MO2) In addition, with each object is associated a well-defined "supplementary" space coordinate $x(t)$, which gives (within experimental errors)the actually observed results of "position measurements"

It is convenient to introduce at this stage a single "representative point" $X(t) = [x_1(t) \ldots x_n(t)]$ in the $3N$-dimensional configuration space, which corresponds to the values of the N supplementary coordinates $x_1(t) \ldots x_n(t)$ in ordinary space. The probability density and the dynamically law of the supplementary coordinates may be expressed in its terms as generalizations of assumptions (1) and (2) of the single–object case:

(MO3) Supposing an adequate normalization, the probability distribution for the representative point X to be located at time t at any point R of $3N$-space is

$$Pr.[X(t) = R] = |\Phi(R,\ t)|^2. \tag{4}$$

(MO4) The i's component $(i = 1 \ldots N)$ of the $3N$-velocity of the representative point $X(t)$ is given by

$$dX_i/dt = m_i^{-1} \nabla_i\, Im Log \Phi(R,\ t)$$

where m_i is the mass of the i's object.

Given an adequate combination of initial conditions for Φ and X, and further assuming (Bell 1982, 1987) that all actual measurements are finally concerned only with observations of positions of things like apparatus pointers and ink marks on paper, the four assumptions listed here give a completely determined theory, in which measurement results are fully specified and guaranteed exactly to reproduce the experimental predictions of ordinary quantum mechanics.

Interpretation

As in the single–object case, it is tempting to try to interpret the single "representative point" $X(t)$ in $3N$-space as the changing positions in 3-space of N classical-like "particles" which are acted upon by a generalized "quantum potential" (Bohm and Hiley 1975). But now this view involves additional complications. For example, in an entangled quantum state of two different objects (e. g. the EPRB set-up), the quantum potential acting on one object may depend on the value (at same t) of the x-coordinate of the other object (Bell 1966). Also, the mutual interdependence of the x-coordinates of two objects may depend on the state of a larger system which includes them both (Bohm and Hiley 1975). It is as if each particle could instantaneously be "informed" about the state of other parts of the universe. Although practical superluminal transmission of information between observers is excluded, this could mean that at the basic underlying level, Nature is not Lorentz-invariant (Bell 1987).

Another objection to such a semi-classical view is that a formulation of the theory in its terms looks artificial and complicated. For example, in a given particular case it is possible to re-formulate assumption MO3 in terms of conditional probabilities for the N separate objects. But the formulation in configuration space terms is surely much more simple and convenient.

We might therefore ask whether the effort to interpret the mathematical "pilot wave" formalism in semi-classical terms is at all worthwhile. After all, this effort might be claimed to be motivated by nothing more than our metaphysical prejudice, conditioned as it is by two and a half centuries of Newtonian mechanics. It is surely remarkable that the "pilot wave" formalism can be interpreted as a "classical" theory capable of reproducing the quantum -mechanical predictions. But it is perhaps even more interesting to try to interpret it as a "quantical" theory, so that it might help in elucidating the problems posed by the usual "collapse" formulation of quantum mechanics.

No–Collapse Theories

In trying to find an alternative interpretation to the many–object "pilot wave" formalism, we should first note that as long as the assumption about $X(t)$ being the actually observed configuration is admitted (along with the auxiliary assumption about practical measurements being finally concerned with positions of things in the laboratory), then no further "interpretation" assumptions are needed in order correctly to account for all the measurement results that we actually get. In this sense, the theory may be regarded as "complete".

Further, we may note that assumption MO1 as given here is actually identical to the basic assumption of Everett's (1957, 1973) "relative–state" formulation of quantum mechanics. As for assumptions MO2 and MO3, these may be regarded as specific answers (not the only ones possible, but reasonable enough) to two problems raised within Everett's formulation - namely, the need to choose a "preferred basis" for the actually observed wave function decomposition into "branches", and the need explicitly to formulate the probabilistic meaning of the "branch measures". And having arrived so far, it also seems reasonable to try adding temporal continuity to the development of the actually observed configuration $X(t)$ by postulating a continuous trajectory, for which the most obvious candidate is the one given by assumption MO4 (Ben-Dov 1990a).

It is true that Everett's formulation is usually identified with a model of "many worlds" which "split" from each other with each quantum measurement (DeWitt 1970), and this model seems to bear little similarity to the "pilot wave" theory as commonly understood. But this "splitting-worlds" scheme was probably not Everett's own view when he suggested his interpretation (Ben-Dov 1990b), and in a way similar to our criticism of the interpretation of the "pilot wave" formalism in terms of classical-like "particles", it may be claimed that the tendency to interpret Everett's formulation in terms of many classical "worlds" instead of a single "quantical" one again reflects remnants of Newtonian metaphysics (Lévy-Leblond 1977).

Thus, as an alternative to such a "classical" reading of the mathematical formulations of the two "no-collapse" theories - Everett's formulation and the "pilot wave" - the following scheme may be suggested. Admitting the four assumptions MO1 - MO4 and the supplementary assumption about measurements finally being of positions, interpret the Schrödinger development of the wave function $\Phi(R,\ t)$ in configuration space as the flow of a conserved $3N$-dimensional " Madelung fluid" of possible configurations, with density $|\Phi(R,\ t)|^2$ and fluid velocity given by Eq. (5) (Ben-Dov 1990a). Now $X(t)$, the single configuration which corresponds to our actual experience, may be regarded as just one typical element carried along by the fluid motions. Thus, we have a well- defined model in which our actual experience consists of perceiving well-localized objects, while the observed measurement results reproduce exactly the experimental predictions of ordinary quantum mechanics.

This still leaves a large space for additional ontology. First, one may hope that the "quantum reality" of possible configurations in $3N$-dimensional configuration space will eventually be integrated into a more general framework, in which relativity theory will also find its natural role. Also, the term "observed" in the description of $X(t)$ is still ontologically unclarified. For practical purposes, it is equated with our actual experience which is immediately given to us, so that in this sense the term is precise. But a complete interpretation should also analyze this term into its constituent ontological elements. Now as the N-object system under discussion includes its own observers, it is clear that any ontological interpretation of the term "observed configuration" should include a reference to the mind-body question, and thus take a position in respect to it.

For example, one may adopt an epiphenomenalist standpoint, and regard the actual awareness of the state of affairs corresponding to the observed configuration $X(t)$ as automatically arising once some set of specific physical conditions (that is, an adequate brain structure) is specified. Still, we may demand whether there exists some quality of "actuality", which distinguishes the single configuration $X(t)$ from all the other Madelung fluid elements, so that actual awareness arises only on $X(t)$ (a "single-mind" view), or whether the possible configurations described by the wave function Φ in $3N$-space all exist on the same footing, each one giving rise to its own states of awareness (a "many-minds" view – see Albert and Loewer 1988).

Another possibility is to assume the independent existence of mind, which "chooses" the actually observed configuration $X(t)$ out of all the possible ones in the Madelung fluid. But in the framework of the theory which includes all four assumptions MO1 - MO4, this "choice" has to be made only once, because assumption MO4 gives a completely determined trajectory for $X(t)$ once that an "initial" configuration $X(0)$ (or for that purpose, a "final" configuration) and the complete evolution of the wave function Φ (determined by the physical Hamiltonian) are specified. Also, this "choice" cannot be effected at a certain time t independently by each local observer for his own degrees of freedom, because actual observations may display non-local correlations in 3-space (as for example in the EPRB set-up), and the "choices" pertaining to the degrees of freedom which represent different observers should reflect these correlations (Stapp 1980). Thus, the relevant space for describing the choice of the actually observed configuration is the $3N$-dimensional configuration space of all the relevant universe. This might suggest, for example, the assumption of a universal Mind, which "chooses" an initial observed configuration $X(0)$ for the N-object system, and then manifest itself through the separate awarenesses of the different observers physically described by the chosen configuration as it evolves in time. Of course, other alternatives may also be possible.

Conclusión

As claimed in the previous section, the many-object "pilot wave" formalism may ontologically be interpreted in several ways, some of which involving assumptions about the observer, rather than about the existence of classical-like "particles". The point is that many ontological schemes may be compatible with the same mathematical formalism, so that the choice between them is to a certain extent a matter of metaphysical taste. Thus, the common belief that the "pilot wave" formalism necessarily implies a semi-classical interpretation of quantum mechanics in terms of well-localized particles "guided" by additional waves is unfounded.

References

Albert D. and Loewer B. . 1988. *Interpreting the Many Worlds Interpretation.* Synthese. December issue.

Bell J. S. . 1966. *On the Problem of Hidden Variables in Quantum Mechanics.* Rev. Mod. Phys. 38. 447-452.

Bell J. S. . 1980. *de Broglie-Bohm, Delayed-Choice Double-Slit Experiment, and Density Matrix.* International Journal of Quantum Chemistry: Quantum Chemistry Symposium 14. 155-159.

Bell J. S. . 1981. *Quantum Mechanics for Cosmologist,* in: *Quantum Gravity 2.* Eds. C. J. Isham; R. Penrose and D. Sciama. (Clarendon Press, Oxford) 611-637.

Bell J. S. . 1982. *On the Impossible Pilot Wave.* Found. of Phys. 12. 989-999.

Bell J. S. . 1987. *Beables for Quantum Field Theory,* in: *Speakable and Unspeakable in Quantum Mechanics.* (Cambridge University Press, Cambridge). 173-180.

Ben-Dov Y. . 1987. *Bell's Version of the "Pilot Wave" Theory.* Fundamenta Scientiae 8. 331-343.

Ben-Dov Y. . 1989. *De Broglie's Causal Interpretation of Quantum Mechanics.* Ann. Fond. Louis de Broglie 14. 343-360.

Ben-Dov Y. . 1990a. *No-Collapse Versions of Quantum Mechanics,* forthcoming in: *The Measurement Problem of Quantum Theory.* Eds. M. Cini and J. -M. Lévy-Leblond (Adam Hilger, Bristol, England).

Ben-Dov Y. . 1990b. *Everett's Theory and the "Many-Worlds" interpretation.* forthcoming Am. Jour Phys.

Bohm D. . 1952. *A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables.* Phys. Rev. 85. 166-179.

Bohm D. and Hiley B. J. . 1975. *On the Intuitive Understanding of Non-Locality as Implied by Quantum Theory.* Found. of Physics 5. 93-109.

Bohm D., Schiller R., and Tiomo J. . 1955. *A Causal Interpretation of the Pauli Equation.* Suppl. al Nuovo Cimento, Serie X, 1. 48.

de Broglie L. . 1927. *La mécanique ondulatoire et la structure atomique de la matiè re et du rayonnement.* Jour. de Phys. et du Rad., Série VI, 8. 225-241.

de Broglie L. . 1930. *Introduction à l'étude de la Mécanique Ondulatoire.* (Hermann, Paris).

DeWitt B. S. . 1970. *Quantum mecahnics and Reality.* Phys. Today 23. No. 9, 30-35.

Einstein A. . 1952: letter to M. Born, May 12, 1952.

Everett H. . III. 1957. *"Relative State" Formulation of Quantum Mechanics.* Rev. Mod. Phys. 29. 454-462.

Everett H. . III. 1973. *The Theory of the Universal Wave Function.* in: *The Many-Worlds Interpretation of Quantum Mechanics.* eds. B. S. DeWitt and N. Graham (Princeton University Press, Princeton, N. J.). 3-140.

Lévy-Leblond J. -M. . 1977. *Towards a Proper Quantum Theory.* in: *Quantum Mechanics, a Half Century Later.* eds. J. Leite Lopes and M. Paty (Ridel, Dordrecht-Holland). 171-206.

Pauli W. . 1928: remark in: *Electrons et Protons, Rapports et Discussions du Cinquième Conseil de Physique.* Institut International de Physique Solvay (Gauthier-Villars, Paris). 280.

Stapp H. P. . 1980. *Locality and Reality.* Found. of Phys. 10. 767-795.